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★**Scale-isometric polytopal graphs in hypercubes and cubic lattices.**

Polytopes in hypercubes and $\mathbb{Z}_n$.

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This is a research monograph, a follow-up to the book [M.-M. Deza and M. Laurent, *Geometry of cuts and metrics*, Springer, Berlin, 1997; MR1460488 (98g:52001)] about l_1-metrics. In recent years a great deal of work has been done on a special case of l_1-metric, namely, the graph distance of the skeleton of finite or infinite polytopes. This monograph identifies polytopes which are combinatorially l_1-embeddable within interesting lists of polytopal graphs.

The monograph consists of 15 chapters and is organized as follows. Chapter 1, a relatively long introduction, gives some basic definitions about graphs, embedding and polytopes. After reading this chapter, any of the other chapters can be read independently. Each of Chapters 2–13 is centered around embeddability for a particular list of graphs. Chapter 2 reports the l_1-status of more than 4000 small fullernesses and their duals, a variety of polyhedra important in chemistry. Chapter 3 considers regular partitions of a Euclidean n-space, n-sphere or hyperbolic n-space. Chapter 4 discusses semi-regular polyhedra and relatives of prisms and antiprisms. Chapter 5 discusses truncations of regular partitions, partial truncations, cappings of Platonic solids, capping of some almost regular l_1-polyhedra on triangular and quadrangular faces, and chamfering of Platonic solids. Chapter 6 shows that exactly 36 of the 92 regular-faced polyhedra are embeddable, with a complete set of illustrating figures. Chapter 7 discusses semi-regular and regular faced n-polytopes for $n \geq 4$. Chapter 8 discusses (r,q)-polycycles, quasi-$(r,3)$-polycycles and other chemically relevant graphs. Chapter 9 reviews plane tiling including 58 embeddable mosaics and other special plane tilings. Chapter 10 presents 28 uniform partitions of 3-space and other special partitions with illustrating tables. Chapter 11 reports the results on the question of whether or not the infinite graph G is embeddable isometrically into a Z_m for some $m \geq n$. Chapter 12 discusses small polyhedra with at most 7 faces and simple polyhedra with at most 8 faces. Chapter 13 discusses bifaced polyhedra. Chapter 14 is about special l_1-graphs. Chapter 15 discusses some generalizations of l_1-embeddings. As a whole, Chapters 2, 4, 5, 6, 12, 13 treat various lists of 3-polytopes. Chapters 9, 10, 11 consider infinite graphs coming from the tiling of $\mathbb{R}^2$, $\mathbb{R}^3$ and from lattices. Chapters 3, 7, 11 consider graphs in $\mathbb{R}^n$. Chapters 14 and 15 consider specifications and generalizations of the notion of embeddability.

Chapters 2, 8, and 11 can be of interest for workers in mathematical chemistry and crystallography.

The authors give concise and independent presentations of most of the topics and the readers of different backgrounds will be able to browse or study those chapters which are of interest for them. The presentation allows the book to serve a variety of needs.

Reviewed by *Ren Ding*