

**MR2429120 (2009e:05086)** 05C10 (05C90 52B70 92E10)**Deza, Michel (F-ENS); Dutour Sikirić, Mathieu (CT-IRBO)****★Geometry of chemical graphs: polycycles and two-faced maps.**

Encyclopedia of Mathematics and its Applications, 119.

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This is a book about the theory of polycycles and two-faced maps. The main motivation for the recent study of these graphs comes from mathematical chemistry. Nevertheless, some of the presented material has a much longer history. Besides classical results, the main part of the material obtained in the last ten years is due to the authors and their coworkers. The main focus of the book is on enumeration, symmetry, extremal properties, face-regularity, metric embeddings, and corresponding algorithmic problems. Along the way several open problems are listed and conjectures posed. The authors used several algorithmic packages, in particular the GAP computer algebra system. The book contains many beautiful drawings that were mostly produced using the program CaGe.

The book is designed such that after reading the first two chapters which give the mathematical background, then the remaining chapters can be accessed (almost) independently. This is particularly welcome since the book is a reference book and not a textbook.

Chapter 3 gives a brief introduction to the fullerenes (defined here more generally than is typically done in chemistry) and serves as a motivating chapter for the rest of the book. Classification of finite fullerenes, toroidal fullerenes, Klein bottle fullerenes, projective fullerenes and plane 3-fullerenes is discussed.

In Chapters 4–8, polycycles are studied, where an  $(r, q)$ -polycycle is a simple plane 2-connected locally finite graph with degree at most  $q$  such that (i) all interior vertices are of degree  $q$  and (ii) all interior faces are  $r$ -gons. Chapter 4 deals with the general notion of the concept and presents several cases that allow classifications. Polycycles on surfaces are also considered. The boundary sequence of a finite polycycle is a sequence of degrees of vertices incident to the exterior face. Chapter 5 handles the unicity of the fillings corresponding to a given boundary sequence. Chapter 6 is concerned with symmetries of polycycles. All possible automorphism groups are listed and all isogonal polycycles (transitive on vertices) and isohedral polycycles (transitive on interior faces) for the elliptic case are given as well as an algorithm for their enumeration. The main purpose of Chapter 7 is to introduce a useful technique of decomposing polycycles into elementary polycycles. The method is very effective in the elliptic case. On the other hand it is proved that for parabolic and hyperbolic parameters  $(r, q)$  there exists a continuum of non-isomorphic elementary  $(r, q)$ -polycycles. Several classifications of elementary polycycles are given. The technique of this chapter is applied in the subsequent chapter on three different problems. One of the problems asks to determine the  $(p, q)$ -polycycles having the maximal number of interior vertices for a fixed number of interior faces. Another problem asks which  $(p, q)$ -polycycles are embeddable into a hypercube with scale 1 or 2.

In the remaining chapters, Chapters 9–19,  $k$ -valent two-faced maps are treated. (Chapter 3 also

pertains to this topic.) More precisely, an  $(\{a, b\}, k)$ -map is a  $k$ -valent map with only  $a$ - and  $b$ -gonal faces, where  $2 \leq a < b$  are given integers. An  $(\{a, b\}, k)$ -map is  $aR_i$  ( $bR_j$ ) if every  $a$ -gonal ( $b$ -gonal) face is adjacent exactly  $i$  times ( $j$  times) to  $a$ -gonal ( $b$ -gonal) faces. An  $(\{a, b\}, k)$ -map is a strictly face-regular map if it is  $aR_i$  and  $bR_j$  for some  $i$  and  $j$ . In Chapter 9, all strictly face-regular maps on spheres and planes are enumerated. Chapter 10 deals with the  $(\{a, b\}, k)$ -spheres that are  $aR_i$  and  $bR_j$  and satisfy  $2k = b(k - 2)$ , that is, the parabolic case. In Chapter 11, weakly face-regular  $(\{a, b\}, 3)$ -maps on spheres and tori are treated. Chapters 12–18 consider spheres and tori that are  $aR_i$ , Frank-Kasper,  $bR_1$ ,  $bR_2$ ,  $bR_3$ ,  $bR_4$ , and  $bR_j$  ( $j \geq 5$ ), respectively. The final chapter treats the so-called icosahedral fulleroids.

In conclusion, the book presents a rich source of chemical graphs (and beyond) and their properties. It should thus serve as a standard reference for researchers in the area.

Reviewed by *Sandi Klavžar*

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