

The Mathematics and Topology of Fullerenes

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VOLUME 4:

THE MATHEMATICS AND TOPOLOGY OF FULLERENES

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 Springer

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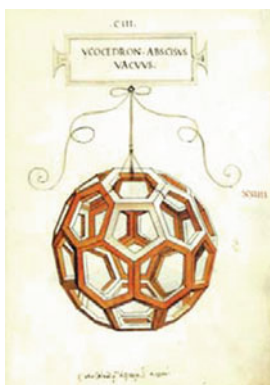
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*to the patient love of Stefania,
Biserka, Floriana*

Foreword



A Quintessential Aspect of the Human Condition – The Appreciation of Beauty in All Its Forms

It is a pleasure to write the foreword for this book probing relationship between mathematics and chemistry as well as computing as I am very conscious of the fact that mathematics made some extremely important and elegant contributions to the story of C_{60} Buckminsterfullerene. Indeed in some ways the C_{60} story epitomizes many aspects of the ways mathematics and science as well as art, architecture and engineering interplay to mix-and-match and inspire sensations of beauty which are at one and the same time complex in deeper aspects and yet simple to appreciate visually. It is not only children that appreciate the elegant beauty of highly symmetric structures such as the one epitomized in the magnificent drawing above by Leonardo. More complex appreciation is engendered in the minds of people who have some familiarity with mathematics which enables them to appreciate the elegance of the general equation governing such structures polyhedral structures:

$$12 = 3 \times n_3 + 2 \times n_4 + 1 \times n_5 + 0 \times n_6 - 1 \times n_7 - 2 \times n_8 \dots$$

In this equation n_m is the number of m -sided polygons in a closed network with trivalent interconnections; when limited to hexagons and pentagons (i.e., only n_5 and n_6 may be non zero) the equation indicates that the network must contain 12 pentagons but there is no limitation on the number of hexagons. Note however that in this case $n_6 \neq 1$. The juxtaposition of Leonardo's drawing with this generalized form of Euler's Equation encapsulates almost the complete spectrum of what it means to be human. The motivation for such a selection of articles as this one reflects perfectly the way in which the patterns of the physical world, often buried deeply in mathematical concepts can be revealed by the combined drives of human curiosity allied with the desire for the cathartic experience of recognizing such patterns for the first time.

Tallahassee, Florida

Harold Kroto

Preface

This book has been conceived during the 24th MATH/CHEM/COMP MCC Conference in Dubrovnik, placed along the beautiful and inspiring coast of Croatia in June 2009, and involves some of the most authoritative experts in this elegant field of nanoscience, placed at the border between mathematics and chemistry.

Since their discovery, fullerenes molecules are intimately connected to mathematics. Geometry, topology, number theory and other mathematical instruments greatly assist researchers to classify fullerenes structures and to predict their unique physical and chemical properties. The name itself of these hollow molecules of pure carbon is rooted in geometry, homage to Richard Buckminster Fuller and to his geodesic dome, the inspiring structure of the “buckminsterfullerene” C_{60} discovered by Robert Curl, Harold Kroto and Richard Smalley in 1985. This epochal, and in somehow epical, discovery has been awarded by 1996 Nobel Prize for Chemistry.

The amplitude of the topics and the level of the contributions are prominent characters of this scientific book that will help researchers in studying and understanding fullerenes properties. For this, we are fully indebted with all authors for their competent and patient works and with Prof. Harold Kroto for his introduction to this monograph that, in many pages, shows that beauty is present in science. We would also like to thank Springer for the opportunity to publish this book and Springer people who allowed all our efforts to become a real text.

Dubrovnik, Croatia

Ante Graovac
Ottorino Ori
Franco Cataldo

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