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Geometry of Cuts and Metrics by M. M. Deza and M. Laurent, Springer-Verlag, Berlin, 1997 ISBN 3-540-61611-X.

This book is definitely a milestone in the literature of integer programming and combinatorial optimization. It draws from the interdisciplinarity of these fields as it gathers methods and results from polytope theory, geometry of numbers, probability theory, design and graph theory around two objects, cuts and metrics.

Deza and Laurent do not only write but with their work actually prove the correctness of the statement. Research on cuts and metrics profits greatly from the variety of subjects where the problems arise. Observations made in different areas by independent authors turn out to be equivalent, facts are not isolated and views from different perspectives provide new interpretations, connections and insights. Every researcher in integer programming and combinatorial optimization will find his fields of research and interest represented in this book. This is one, but not the only aspect that makes the book unique.

The book has five parts, each of which is fairly self-contained.

Part 1 treats relations between cuts and metrics. Every generator of the cut cone (the generators of the cut cone are all incidence vectors of cuts of a given graph) defines a semimetric, i.e., a symmetric function on the pairs of vertices, satisfying the triangle inequalities and f(i,i) = 0 for all vertices i. (Of course, not every semimetric is a cut.) Of major interest in this part are the characterizations of cuts by means of measure theory and l_1-embeddability including, in particular, the following theorem: a semimetric belongs to the cut cone if and only if it is isometrically l_1-embeddable.

Part 2 studies so-called hypermetric spaces. Hypermetric inequalities are inequalities of the form

$\sum_{1\leq i<j\leq n} b_i b_j d(i,j) \leq 0$ with $b \in \mathbf{Z^n}$, $\sum_{i=1}^{n} b_i = 1$.

One can prove that every semimetric in the cut cone satisfies the family of hypermetric inequalities, yet not every semimetric satisfying the family of hypermetric inequalities is a member of the cut cone. Hypermetric spaces, the hypermetric cone and the connections to point lattices and Delaunay polytopes are the central issue in Part 2.

Part 3 is devoted to investigations of graphs whose path metric is l_1-embeddable or hypercube-embeddable. It is shown in the book that a graph is l_1-embeddable if and only if a non-negative multiple of its path metric is hypercube-embeddable. Of particular beauty is the fact that l_1-embeddable graphs can be recognized in polynomial time.

Part 3 is directly connected to Part 4 of the book that treats questions of the form: given a distance functionon a finite number of points, decide whether this distance function is hypercube-embeddable. There are some distance functions for which this problem is easy to solve. For others, the decision about hypercube-embeddability is NP-hard. For various other classes of metrics, there are conditions available that can be tested in polynomial time and ensure hypercube-embeddability.

Part 5 deals with the geometry of the cut cone and the cut polytope. It surveys extensively polyhedral material, including the fundamental facet-manipulating operations such as switching, the family of triangle inequalities and more general hypermetric inequalities. Very appealing is the detour to cycle polyhedra of binary matroids and the questions that the authors discuss in this context about linear relaxations by the triangle inequalities and Hilbert bases. Also very interesting are the discussions about the completion problem and the connections to geometric questions such as the partitioning of a set in the n-dimensional space into $n+1$ sets of smaller diameter.

The book is very nicely written, although it is quite dense and requires a lot of knowledge to understand the details. Starting with the important definitions that it resorts to, each of the chapters is self-contained. I found it helpful to read Chapter 1, the outline of the book, in the beginning.

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It really helps in getting through the advanced parts. The book is also very well structured. With knowledge about the relevant terms, one can enjoy special subsections without being entirely familiar with the rest of the chapter. This makes it not only an interesting research book but even a dictionary. The material is up-to-date, and there are various sections that contain enough open questions fora couple of Ph.D. theses.

In my opinion, the book is a beautiful piece of work. The longer one works with it, the more beautiful it becomes. - ROBERT WEISMANTEL, BERLIN

Lectures on Polytopes by G. Ziegler,
Springer-Verlag, Berlin, 1995 ISBN 0-387-94329-3

During the last 30 years, the theory of (convex) polytopes has drawn growing attention. As the convex hull of finite point sets in euclidean spaces, polytopes are very natural objects; therefore, it is not surprising that they have a great number of applications in such diverse mathematical areas as Linear and Combinatorial Optimization, Functional Analysis, AlgebraicGeometry and Semialgebraic Geometry. This book does not concentrate so much on these fields of applications as on the theory of polytopes itself, which has by now obtained an enormous scope and depth. The reader, however, will still find numerous references to related areas.

A very motivating and example-oriented introduction is presented in Chapter 0, which gives the reader a first impression of the interesting subject and introduces the basic terminology at the same time. This chapter explains in detail

the different ways of representing polytopes which are important in Computational Geometry and Optimization. Chapters 1 and 2 present the foundations of convex geometry and the most important facts about face lattices of polytopes. Chapter 3 studies the edge graphs of polytopes and extensively discusses the newest results on the diameter of such graphs. These are of particular importance for Linear Optimization as they reflect the worst possible behavior of best possible edge-following LP-algorithms.
This chapter also includes Kalai's extremely elegant proof of the fact that the edge-graph of a simple polytope already determines its complete face lattice. The edge-graphs of 3-dimensional polytopes are characterized by planarity and 3-connectedness. This is the famous theorem of Steinitz which is the basis for many further results about 3-dimensional polytopes. A new proof of this theorem is presented in Chapter 4. This proof is based on a graph reduction technique due to Truemper, and it avoids some of the complications of earlier proofs.
The two following chapters are devoted to realizability problems for higher-dimensional polytopes. In analogy to the theorem of Steinitz, the question is whether cell-complexes with given geometric or combinatorial properties are isomorphic to the face-lattice of polytopes. For such problems, oriented matroids and Gale-diagrams have proven very useful. As an application of this theory, the reader is presented with a 5-dimensional polytope which has a 2-dimensional face whose shape cannot be arbitrarily preassigned. Meanwhile, Richter-Gebert have constructed a 4-polytope with this property, thereby solving a problem posed in the book.
The part of the theory of oriented matroids that is needed in polytope theory is described very well. In Chapter 7, this theory is studied in depth and is applied to zonotopes and other objects related to polytopes like arrangements of hyperplanes and tilings of space.
Chapter 8 introduces the spectacular results on the numbers of faces of polytopes, the "Upper-Bound-Theorem" and the "g-Theorem". The concept of shellability and the related h-vectors, which can be defined by it, are essential for these results. Both are explained in detail and applied to the first construction of a polytope having a partial shelling which cannot be extended to a complete shelling.
The last chapter studies fiber polytopes which are important for Grbner bases. As an application, the author presents a construction of the permuto-associahedron. The book ends with an extensive list of references.

All chapters contain a useful collection of problems, beginning with "warm-ups" and ending with important open problems. The book excels because of its lucid presentation, which is supported by many helpful illustrations. The careful descriptions of the results provide an excellent motivation for students and make the book a valuable basis for a course on polytopes. The publication of the book has obviously led to the solution of some of the open problems described in it.

The reader will be delighted to find that the author has established a web site (http://winnie.math.tu-berlin.de/ziegler) which, in addition to the correction of minor errors, has all the information on these interesting new developments. These updates will be continued in a revised edition to appear soon. As the book contains all important techniques of polytope theory and also many new results, it is most useful both for the expert and for other mathematicians and computer scientists who use polytopes in one of the application areas mentioned.

I very much enjoyed reading it. - PETER KLEINSCHMIDT, PASSAU

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