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Scale-isometric polytopal graphs in hypercubes and cubic lattices. Polytopes in hypercubes and Z_n. (English)

London: Imperial College Press. ix, 175 p. £ 28.00 (2004). ISBN 1-86094-421-3/hbk

This is a fascinating book on a topic with surprising applications. The basic idea is that many graphs derived from polytopes can be scale-isometrically embedded in a hypercube. For instance, a tetrahedron can be embedded in the unit cube with all pairs of vertices at a Euclidean distance of $\sqrt{2}$ – or, more interestingly, at a l^1 distance of 2. This can be extended to many other polytopes and space tilings.

There are significant applications to computational chemistry, as such an embedding gives a useful way to represent the intrinsic structure of large molecules such as fullerenes. This book is a detailed study of this topic. It's rather strong on listings and classifications, and (perhaps inevitably) somewhat short on big theories. This makes it accessible to the curious, as well as for anybody needing some one result. Embeddings are studied for regular and semiregular polytopes, plane and space tilings, small non-regular polyhedra, and other structures.

The book is generously illustrated, although the quality of the illustrations is very variable. Fig. 13.1, for instance, appears to have been faxed, with the result that some labels are utterly unreadable; others (e.g., Fig. 6.1) have dark "halos" that suggest that they were recreated from overcompressed JPEG files. Most illustrations are excellent.

There are rather a lot of minor grammatical errors; authors taking the trouble to write in a second language deserve better editorial support! However, it does not seem to the reviewer that syntax such as "The following embeddings are true $Z^\infty \to H^\infty \subset Z^\infty$" (page 23) is good mathematical writing in any language. Embeddings exist (or don't); they are not "true". While the writing may occasionally be annoying, it is for the most part unlikely to confuse the reader.

It should also be noted that cubical complexes were not introduced by Novikov in 1986 (as stated on page 20), but go back (at least) to Kan in the 1950's. Nonetheless, despite its minor weaknesses, this book could be read for pleasure by anybody with some familarity with polytopes, and will be very useful to a smaller group.

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Keywords : scale-isometric embedding; polytopal graph; hypercube; integer lattice

Classification :

- ∗52B11 n-dimensional polytopes
- 52-02 Research monographs (convex and discrete geometry)
- 05-02 Research monographs (combinatorics)
- 05C12 Distance in graphs